

Slide 1 – Laboratory Math I: Exponents, Units and Scientific Notation

Welcome to the National Institutes of Health, Office of Intramural Training & Education's Webinar on Laboratory Math 1: Exponents, Units and Scientific Notation. This Webinar is intended to review some basic math skills that are commonly used in biomedical research. Even if you have a strong mathematic background, it may serve as a good refresher.

Slide 2 – What is Lab Math?

The first question to address is simply, What is lab math? Lab math is any math skill or technique that is used in laboratory science. It can range from basic math skills, to complex calculus and trigonometry. In this Webinar, we will review the basic math skills you will need to understand to perform many of the calculations common to biomedical research. Because much of scientific research is performed on either microscales or macroscales, we will cover multiplication and division with exponents, focusing on working with the powers of ten. We will discuss scientific notation and how its use can simplify calculations and the presentation of data.

We will also introduce the mathematics of the metric system, cover the common nomenclature, and demonstrate how to convert the units within the system.

In a future Webinar we will discuss making solutions and dilutions in the lab.

Slide 3 – Exponents

Before we can discuss how to multiply and divide exponents, it's best to define what an exponent is and cover a couple of the basic rules.

Simply defined, an exponent is a shorthand notation for the number of times a number is multiplied by itself.

For example, n to the fourth power is shorthand for n times n times n times n , or n times itself four times. n to the second power, or n squared, is equal to n times n , or n times itself twice.

Exponents can also be negative numbers. Negative exponents are shorthand notation for the inverse of the corresponding positive exponents. Therefore, n to the negative fourth power is equal to one divided n to the fourth. This extrapolates out to be one divided by n times itself four times.

By rule then, n to the first power is just n . Conversely, anytime you see a number, in this case denoted by the letter n , it can be written in an exponent as n to the first power.

Also note that any number to the zero power is equal to one. So, n to the zero power is 1. Conversely, you can denote 1 as any number to the zero power.

Slide 4 – Powers of Ten

As mentioned earlier, much of biomedical research is performed on micro- and macroscales. These numbers can be long and cumbersome both to denote and to work with. To help simplify these numbers and make them more “workable”, many numbers in a research setting are written as powers of ten.

As with all exponents, in powers of ten, the exponent is equal to the number of times that 10 is multiplied by itself. However, the base number ten is easy to work with as the exponent is also equal to the number of places to the right of the “1” that you place the decimal point. Said another way, the exponent is equal to the number of zeros between the “1” and the decimal place.

In 10 to the zero power, there are no zeros between the “1” and the decimal point. In 10 to the power of one, there is one zero between the “1” and the decimal point. The same is true with 10 to the second power or 10 squared. It is equal to 10 times 10 which is 100. And in one hundred, there are two zeros between the “1” and the decimal point. As you start to get to larger and larger numbers, you can more easily understand why using the short hand exponent is so appealing. It is easier to write and work with 10 to the sixth power, than to write out the “1” and the six zeros between the “1” and the decimal point.

Slide 5 – Negative Powers of Ten

Just as powers of ten can be used as shorthand for very large numbers, they can also be used as shorthand for very small numbers that use up a lot of space when written out. We mentioned earlier that negative exponents are equal to the inverse of the corresponding positive exponents. So for powers of ten, a negative exponent is equal to the number of spaces to the left of the “1” that you place the decimal point. So for ten to the negative first power, you will move the decimal point one place to the left of the “1” and your answer will be zero point one, or one tenth. Ten to the negative second power would be equal to one divided by 10 to the second power or one divided by one hundred, also called one one hundredth and written zero point zero one. Notice, the decimal point is two places to the left of the “1”. As a general rule, it is best to use a leading zero when writing a number less than one. That is, there should be a zero to the left of the decimal place to indicate that the number is less than one because the decimal point may be hard to see.

Again, when you start to work with very small numbers, such as one one millionth, it’s beneficial to use the shorthand of ten to the negative sixth power.

Slide 6 – Numbers Written as Powers of Ten

Any number can be written as a power of ten. As we will discuss in future slides, this becomes a powerful tool for simplifying the calculations that are needed for lab math.

To write a number as a power of ten, it is easiest to start with the number itself and write it with a decimal place present, even if one isn't there already. So "3" is written "3.0". The next step would be to determine how many places you need to move your decimal point such that only one integer remains to the left of the decimal point. In this case, zero. The number of places you have to move your decimal point then becomes the exponent on the ten. Remember that 10 to the zero power is equal to one. Since any number multiplied by one is equal to the number itself, we can write the number "3" as 3.0 times 10 to the zero power. For a number like 30, we write the number as 30.0. Then we move the decimal point to the left one place. One then becomes the exponent of the ten. So, we can write "30" as 3.0 times 10 to the first power. 300 first is written as 300.0. We move the decimal point to the left two places and 300.0 can now be written as 3.0 times 10 to the second power.

You can use the same principle for writing numbers less than one as well. The only difference is you will be moving the decimal place to the right, and the number of places you move the decimal point becomes the negative exponent of the ten. For example, 0.004. We will move the decimal point three places to the right to get to 4.0. Therefore, the exponent on the ten will be negative three. And 0.004 can be written as 4.0 times ten to the negative third power.

Take some time to work out the rest of the examples.

Now check your answer. If you got them right, great. Good job. If not, try reviewing this slide again. If you are having trouble understanding the purpose of writing numbers in powers of ten, take for example one of the most famous numbers in science. Avogadro's number for defining the number of molecules in a mole of any substance is 6.0225 times ten to the 23. That is a lot of zeroes to write out if you don't use shorthand to do it. Partly for this reason writing numbers as powers of ten is called scientific notation.

Slide 7 – Scientific Notation

Scientific Notation is a method of writing numbers in terms of decimal numbers between one and nine multiplied by a power of ten. It dictates that there is exactly one place holder to the left of the decimal point, an integer between 1 and 9. The rest of the number is written after the decimal point and then multiplied by the appropriate power of ten. This is most easily explained by looking at examples of numbers written in scientific notation.

The number one point two five is easy to translate into scientific notation. It already meets many of the criteria. There is one place holder to the left of the decimal point. In this case, the integer one. To convert it to scientific notation, we just have to multiply it by the appropriate power of ten. Since we

want the number to stay the same, we multiply by one, which is 10 to the power of zero. As a note, you will almost never see 10 to the power of zero written out unless it is being used in the arithmetic.

What about converting 25.78 into scientific notation? Can you do it? (Pause) If you follow the rules and have only one place holder to the left of the decimal point, it would mean moving the decimal point one place to the left. As we learned in previous slides, that one becomes the exponent of our power of ten, so 25.78 is equal to 2.578 times ten to the first power. Now, what about one hundred thousand, four hundred ten? Can you convert it to scientific notation? (Pause) Did you come up with 1.0041 times ten to the fifth? If so, good job. If not, try reviewing the criteria for scientific notation and then review the principles from the previous slide.

One last note about scientific notation: When writing a number in scientific notation, you only record significant digits. Which raises the question, "What is a significant digit?" One of the most important aspects of your interactions is to monitor the time. Remember that time is a resource valued by most everyone. So, mind the clock. If you are presenting, stay within your scheduled time allotment. You will find many scientists do not do this. So, by being respectful of the time limit, you will stand out from the crowd in a good way. In the same light, do not monopolize a presenter's time. If you are viewing a poster, stay on topic. Discuss the poster material and the project. If your conversation takes you to another topic, ask to follow up with the presenter later. This frees that presenter up to interact with more people who want to discuss his or her poster as well as gives you an opportunity to network outside the scheduled meeting sessions.

Slide 8 – Identifying Significant Digits in a Number

A significant digit is any digit within a number that contributes to the precision of the number. More simply put, all digits are significant except for placeholders. Again, the easiest way to understand what significant digits are is by example.

So, if any digit in a number other than placeholder "zeroes" is a significant digit, what is a placeholder zero? Basically, any zero or series of zeros that are not between two integers. So, in the number zero point zero zero zero five, all the zeros are placeholders. So, which zeros are place holders in the number thirty five thousand? (Pause) All of them. Now, is the zero in the number one thousand fifty four a placeholder? (pause) No, it is not because there are integers on either side of it. Therefore, that zero is significant.

Another way to gauge whether a digit is significant is to determine if you would report it in scientific notation. Placeholder zeroes of a number are represented by the powers of ten in scientific notation. Therefore, any zero that you would NOT report in scientific notation is a placeholder and thus is not significant.

An important rule when multiplying and dividing numbers is to remember that your result should have as many significant digits as the measured number with the fewest significant digits. So if you multiply a

number with two significant digits by a number with seven significant digits, your result will have only two significant digits.

Slide 9 – Significant Digit Examples

Let's go through a few examples and determine the significant digits of some numbers. The number twenty five hundred has two significant digits. The two and the five are both significant. The two zeroes are both placeholders. This makes them not significant.

What about the number 0.0000654? How many significant digits are there and which digits are they? (Pause) There are three significant digits: the six, the five, and the four are all significant. Again, in this example the zeroes are placeholders. As practice, how would you write this number in scientific notation? (Pause) 6.54 times ten to the -6.

Now, how many significant digits are in the number one thousand forty five point zero one? (Pause) All six of the digits are significant as none of the zeros are placeholders. Again, if you were to write this number in scientific notation, you would use all the digits. It would be one point zero four five zero one times ten to the third.

Slide 10 – Multiplying Powers of Ten

Now we are going to look at how to perform calculations with powers of ten.

When multiplying powers of ten, you add the exponents together and the result is ten to the sum of the individual exponents.

Let's look at a few examples.

We multiply ten to the second power by ten to the third power by adding the exponents, two and three, and keeping the base ten. This yields us ten to the fifth power. As a manner of checking the math, we can use the full written out numbers of 100 (ten to the second power) times 1000 (ten to the third power), which is equal to one hundred thousand, the same as ten to the fifth power.

Now, what would be the result of multiplying ten to the fifth power and ten to the fourth power? (Pause) Following the rule of adding the exponents, we would get ten to the power of five plus four, or ten to the ninth power.

The principle is the same for negative exponents. Ten to the negative four times ten to the third is equal to ten to the sum of negative four and three, which is negative one.

What about ten to the fifth times ten to the negative two? (Pause) Did you come up with ten to the third? Feel free to check these using your calculator.

Slide 11 – Dividing Powers of Ten

Logically, division is the opposite of multiplication. Practically speaking, when dividing powers of ten, it is the opposite of multiplying powers of ten. That is, when dividing powers of ten you subtract the denominator exponent from the numerator exponent to reach the result.

Again, let's look at a few examples to understand how this works.

We divide ten to the fifth power by ten to the third power by subtracting the exponents, three from five, and keeping the base ten. This yields us ten to the second power.

Now, what would be the result of dividing ten to the seventh power by ten to the ninth power? (Pause) Following the rule of subtracting the exponents, we would get ten to the power of seven minus nine, or ten to the negative second power.

Another tool for dividing powers of ten, is to realize that dividing by a power of ten is the same as multiplying by the negative of the bottom exponent. This is basically saying that the one over ten to the ninth is equal to ten to the negative ninth. So by basic mathematic principles, ten to the seventh divided by ten to the ninth is equal to ten to seventh multiplied by one over ten to the ninth. This is then equal to ten to the seventh times ten to the negative ninth. Knowing that we can multiply powers of ten by adding their exponents, we get ten to the seven plus negative nine, or ten to the negative second power.

Now using this technique, can you divide ten to the third by ten to the thirteenth power? (Pause) This is equal to ten to the third multiplied by ten to the negative thirteenth, which is equal to ten to the power of three plus negative thirteen, or ten to the negative tenth power.

Slide 12 – Working with Numbers in Scientific Notation

Now that we understand how to work with the powers of ten on their own, we need to learn how to work with numbers in scientific notation. That is, a base number times a power of ten.

The first rule to know is that when adding or subtracting numbers in scientific notation, the numbers must be in the same power of ten. We will discuss examples of this in the next slide, but two basic steps must be taken when adding or subtracting numbers in scientific notation.

First, convert the smaller number into the same power of ten as the larger number. Remember the rules of moving the decimal place and adding to or subtracting from the exponent. Once in the same power of ten, add or subtract the root numbers while leaving the result in the same power of ten as the numbers being added or subtracted.

To multiply and divide numbers in scientific notation, you multiply or divide both the root numbers AND the powers of ten. So when multiplying, you would multiply the root numbers AND add the exponents

together. When dividing you would divide the root numbers AND subtract the exponents. We will cover examples of this in later slides.

Slide 13 – Working with Numbers in Scientific Notation (Pt2)

As stated in the previous slide, to add or subtract numbers in scientific notation the numbers must be in the same power of ten. Once, in the same power of ten, you simply add the root numbers together. So, lets look at a few examples.

Consider three point five times ten to the third plus four point zero times ten to the fourth. The first thing we need to do is convert the smaller number (3.5×10^3) into a number with the power of ten to the fourth. Since we would be adding one to the exponent, we would then move the decimal point one place to the left. So our problem would now read zero point three five times ten to the fourth plus four point zero times ten to the fourth. Now we just add the root numbers (four point zero and zero point three five) together and we get four point three five times ten to the fourth.

Slide 14 – Working with Numbers in Scientific Notation (Pt2)

Now see if you understand the principles of adding numbers in scientific notation by working out this problem. Five point one one times ten to the fifth power plus six point five times ten to the second power. (Pause)

First, convert the smaller number (6.5×10^2) to a power of ten to the fifth. Since you will be adding three to the exponent, it means you move the decimal point three places to the left and it gives you zero point zero, zero, six, five times ten to the fifth. Now, you just add to the two root numbers together to get five point one one six five times ten to the fifth power.

Slide 15 – Working with Numbers in Scientific Notation (Pt3)

Multiplication and division of numbers in scientific notation is a little more straightforward than addition and subtraction. There is no need to convert the numbers to the same power of ten. To multiply or divide powers of ten, you treat the root number and the powers of ten individually.

Lets start with multiplication. First, you multiply the root numbers together and then you multiply the base ten powers, which means you add the exponents.

Again, let's go through a few examples:

Two times ten to the fourth multiplied by four times ten to the third. Start by multiplying the root numbers two and four. Then multiply the powers of ten by adding the exponents four and three. This gives you a result of eight times ten the seventh.

Now you work out five times ten to the negative third multiplied by two times ten to the fifth. (Pause) Did you start by multiplying the root numbers five and two together? Then did you multiply the powers of ten by adding the exponents? If so, hopefully you came out with a result of ten times ten to the second. Hopefully you also remember to write your answer in scientific notation by moving the decimal place to the left one place and adding one to the exponent to give the final answer of one point zero times ten to the third power.

Now work through 1.25 times ten to the fourth multiplied by three point five times ten to the sixth. Remember what you have learned about significant digits when reporting your answer. (Pause) Did you end up reporting four point four times ten to the tenth power? Remember that you only report as many significant digits as the measured number with the fewest significant digits. In this case three point five times ten to the sixth only has two significant digits and thus your result can only have two significant digits.

Slide 16 – The Metric System

Having come this far in the Webinar, you may be asking why these basic math skills are considered laboratory math. The answer is they are all required for working with the metric system, and all scientific measurements are made using the metric system.

So what is the metric system? It is a decimal system of weights and measures. Since there has been a large focus on the powers of ten, it will probably not surprise you that “decimal” means pertaining to powers of ten. In the next few slides we will introduce the units of the metric system and how the powers of ten make it easier to work with scientific measurements.

First it is important to know the base units for each measurement. The name “Metric” is derived from the unit of measurement for length, the meter. The unit of measurement for mass is the gram, for volume it is the liter, for time it’s the second, and the unit of measure for the amount or total number of something is the mole.

Slide 17 – Working in the Metric System

Knowing how to work within the metric system requires knowing the terminology, understanding what each term means, and appreciating how they relate to one another. We are going to use length in our discussions of how the metric system works, but the same approach would apply to mass or volume or any other unit.

The base unit for any measurement is one times ten to the zero power, or 1. One meter is the base unit for a measure of length and can be written one times ten to the zero meters.

The commonly used prefixes in the metric system refer to increases or decreases in values by 1000, or ten to the third power.

“Kilo” means 1000. Thus, one kilometer is equal to 1000 meters or one times ten to the third meters.

“Milli” means $1/1000$. Thus, one millimeter is equal to $1/1000$ of a meter or one times ten to the negative third.

These prefixes allow you to further use shorthand to limit your writing. It is easier to write 15 millimeters than to write out one point five times ten to the negative two meters. It is important to note that because the common prefixes represent shifts of 1000 or ten to the third that not all numbers will fall between 1 and 9. It is acceptable to write 15 millimeters as opposed to its scientific notation in the base unit.

Slide 18 – Prefixes in the Metric System (Pt1)

Now that we have worked through the process of converting up and down the metric scale, try and fill in the numbers associated with each prefix. The first few are done for you.

As you start to complete the table, you may notice a short cut. To move from the base to kilo, you simply add three to the exponent and make it one times ten to the third. The same can be done when moving to the smaller units. You simply subtract three from the exponent. To move from the base to milli, you would subtract three from the exponent and get one times ten to the negative third power.

Slide 19 – Prefixes in the Metric System (Pt2)

Now to move up from kilo to Mega, you add three more and Mega is equal to one times ten to the sixth. The same is done to get one times ten to the ninth for Giga and one times ten to the twelfth for Terra.

If you go from milli to micro you subtract another three, you see that micro is one times ten to the negative six. If you continue you will have nano equal to one times ten to the negative nine and pico equal to one times ten to the negative twelve.

These are merely the most common prefixes. There are many more for both larger and smaller numbers. Also, in some systems of measurement additional prefixes are used to represent the numbers other than orders of 1000. An example would be the centimeter, used to represent one one hundredth of a meter or one times ten to the negative second meters.

Something else that is important to note: If one nanometer is equal to one times ten to the negative ninth meters, the inverse is true as well. This means that one meter is equal to one times ten to the ninth nanometers. Logically that makes sense. If a nanometer is much smaller than a meter, there should be a lot of nanometers in every meter. In fact, there are one times ten to the ninth nanometers in every meter.

Slide 20 – Working with the Metric System (Pt1)

Now that we know the terminology for prefixes within the metric system and to what number they correspond, let's look at how to convert between them mathematically. Specifically, let's calculate how many meters there are in the other units of length.

A meter is base unit, meaning one times ten to the zero power meters, or one meter.

A kilometer is one times ten to the third meters. We could also say that there are ten to the third meters in a km.

We also know that a Megameter is one times ten to the third kilometers and thus there are ten to the third kilometers per Megameter. To determine how many meters are in a megameter, you multiply ten to the third kilometers per megameter times ten the third meters per kilometer. The kilometers cancel out, the exponents are added and you get one times ten to the six meters per megameter.

Slide 21 – Working with the Metric System (Pt2)

We can use the same principles to work into the smaller units as well.

We know that one millimeter is equal to one times ten to the negative third meters, or that there are ten to the negative third meters per millimeter.

We also know that a micrometer is one times ten to the third millimeters or that there is ten to the negative third millimeters per micrometer. Based on what we learned in the last slide, can you then determine how many meters there are in a micrometer? (Pause)

If there are ten to the negative third millimeters per micrometer and ten to the negative third meters per millimeters, if you multiply these together the millimeters will cancel out, and you can add the exponents. Again, getting ten to the negative three plus negative three, or ten to the negative six meters per micrometer.

Slide 22 – Working with the Metric System (Pt3)

This principle is important when converting between units that do not start with the base unit. For example: 5 kilometers is equal to how many nanometers?

We know that there are ten to the third meters per kilometer. And we also know that there are ten to the ninth nanometers per meter. Therefore we can first multiply five kilometers by ten to the third meters per kilometers and then multiply that by ten to the ninth nanometers per meter. The kilometers cancel out in the first multiplication step. The meters cancel out in the second multiplication step and we solve that five kilometers is equal to five times ten to the twelfth nanometers.

Now, try and work out how many kilometers are in 15 millimeters.

The first step is to the set up the equation.

15 millimeters is equal to how many kilometers? (Pause)

Then, write out what you know about each in terms of meters. There are ten to the negative third meters per millimeter. And there are ten to the negative third kilometers per meter. Thus, we can multiply 15 millimeters by ten to the negative third meters per millimeter and then multiply that by ten to the negative third kilometers per meter. The millimeters cancel out in step one, the meters cancel out in step two and we are left with fifteen times ten to the negative sixth kilometers.

Slide 23 – References

Thank you for watching this Webinar on Lab Math. We hope it has helped you understand the fundamental skills needed to perform many of the calculations you will face in the lab.

For more training resources, please visit our website by clicking the link on this slide.

Also, check back on that page in the near future for Lab Math II: Solutions and Dilutions.